Discontinuous Galerkin Methods for Nonlinear Scalar Hyperbolic Conservation Laws: Divided Difference Estimates and Accuracy Enhancement

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In this paper, an analysis of the accuracy-enhancement for the discontinuous Galerkin (DG) method applied to one-dimensional scalar nonlinear hyperbolic conservation laws is carried out. This requires analyzing the divided difference of the errors for the DG solution. We therefore first prove that the α-th order $(1 < \alpha < k+1)$ divided difference of the DG error in the L^2 norm is of order $k+\frac{3}{2}-\frac{\alpha}{2}$ when upwind fluxes are used, under the condition that $|f'(u)|$ possesses a uniform positive lower bound. By the duality argument, we then derive super- convergence results of order $2k+\frac{3}{2}-\frac{\alpha}{2}$ in the negative-order norm, demonstrating that it is possible to extend the Smoothness-Increasing Accuracy-Conserving filter to nonlinear conservation laws to obtain at least $\left(\frac{3k}{2}+1\right)$ th order superconvergence for post-processed solutions. As a by-product, for variable coefficient hyperbolic equations, we provide an explicit proof for optimal convergence results of order $k+1$ in the L^2 norm for the divided differences of DG errors and thus $(2k+1)$ th order superconvergence in negative-order norm holds. Numerical experiments are given that confirm the theoretical results.